

Islamic University of Science & Technology

Department of Mathematical Sciences

Syllabi for Ph.D Entrance Test

Part A: Research Methodology

Maximum Marks: 35

Part B: Statistics

Maximum Marks: 35

Part C: Mathematics

Maximum Marks: 35

Note: Part A is compulsory for both Statistics and Mathematics Students. While as Part B is for Statistics students and part C for Mathematics students

Part A: Research Methodology

Section I

Research Fundamentals, Aims and objectives of research, Types of research: basic, novel and applied research. Tools for searching a Research topic: books, journals, internet, discussions etc. Research hypothesis; steps in research design. Quantitative and qualitative methodology hypotheses. Publication of research, plagiarism, Intellectual property rights. Quality of research work. Papers indexing, impact factor, citation index, H index. Web resources: E-Journal access, UGC-info net, E-books, Search engines, Google Scholar, Scopus, SciF finder, Math SciNet

Section II

Statistics a conceptual frame work, Statistical enquiry, collection of data, classification and tabulation of data. Diagrammatic and Graphic presentation of data.

Measures of central tendency: mean, median, mode. Measures of dispersion-range, mean deviation, quartile deviation Standard deviation and variance. Measure of skewness- Karl-Pearson's and Bowley's methods. Measures of Kurtosis.

Hypothesis testing, general procedure and errors in hypothesis testing, hypothesis testing for population parameters with large and small samples, Hypothesis testing based on F-distribution and t-distribution. Chi-Square test for goodness of fit, chi-square test for population variances, chi-square test for association

Section III

Elementary set theory, Real Number system, supremum, infimum, Continuity, Uniform continuity, Differentiability, Functions of several variables, partial derivative, Rules for

differentiation, derivatives of common functions, maxima, minima, Integration. Vector spaces, subspaces, linear dependence, basis, dimension, algebra of linear transformations. Algebra of matrices, rank and determinant of matrices, linear equations. Eigen values and eigenvectors. Matrix representation of linear transformations. Change of basis, canonical forms, diagonal forms, and triangular forms.

Section IV

Logical Reasoning, Graphical Analysis, Analytical and numerical Ability, Quantitative Comparisons, Series Formation, Puzzles etc.

Part B: Statistics

Section I

Probability: Sample space, discrete probability, independent events, Bayes theorem. Random variables and distribution functions (univariate and multivariate); expectation and moments. Independent random variables, marginal and conditional distributions. Characteristic functions, Moment generating functions and probability generating functions. Probability inequalities (Tchebyshef, Markov, Jensen). Modes of convergence, weak and strong laws of large numbers, Central Limit theorems (independent case).

Probability Distributions: Standard discrete and continuous univariate distributions. Sampling distributions. Standard errors and asymptotic distributions, distribution of order statistics and range. Compound, truncated and mixture distributions.

Section II

Statistical Inference: Methods of estimation. Properties of estimators. Confidence intervals. Tests of hypotheses: most powerful and uniformly most powerful tests, Large sample inference, Likelihood ratio tests. Analysis of discrete data and chi-square test of goodness of fit. Large sample tests.

Simple nonparametric tests for one and two sample problems, rank correlation and test for independence. Elementary Bayesian inference.

Multivariate Analysis: Multivariate normal distribution, Wishart distribution and their properties. Distribution of quadratic forms. Inference for parameters, partial and multiple correlation coefficients and related tests. Data reduction techniques: Principle component analysis, Discriminant analysis, Cluster analysis, Canonical correlation.

Section III

Linear Models\Regression: Gauss-Markov models, estimability of parameters, Best linear unbiased estimators, tests for linear hypotheses and confidence intervals. Analysis of variance and covariance. Fixed, random and mixed effects models. Simple and multiple linear regression. Elementary regression diagnostics. Logistic regression.

Stochastic Processes: Markov chains with finite and countable state space, classification of states, limiting behaviour of n -step transition probabilities, stationary distribution. Poisson process, birth process, birth-death process

Reliability & Survival Analysis: Series and parallel systems, hazard function and failure rates, censoring and life testing. Kaplan–Meier estimators, Cox proportional hazards model. Comparison of survival functions. Competing risks model.

Section IV

Sampling: Simple random sampling, stratified sampling and systematic sampling. Probability proportional to size sampling. Ratio and regression methods.

Design of Experiments: Completely randomized, randomized blocks and Latin-square designs. Connected, complete and orthogonal block designs, BIBD. 2^k factorial experiments: confounding and construction.

Linear Programming: Linear programming problem and its formulation; graphical method, simplex method, revised simplex method. Transportation Problems, Methods of obtaining initial basic feasible solutions, MODI method. Assignment problems. Sequencing problems: problems with n jobs and 2 machines, problems with n jobs and k machines. Games and Strategies: Two person zero-sum games, Maximin-Minimax Principle, Mixed Strategies, Solution of 2×2 and $m \times n$ games.

Part C: Mathematics

Section I

Analysis: Real functions; limit, continuity, differentiability; sequences; series; uniform convergence; functions of complex variables; analytic functions, complex integration; singularities, power and Laurent series; metric spaces; stereographic projection; topology, compactness, connectedness; normed linear spaces, inner product spaces; dual spaces, linear operators; Lebesgue measure and integration; convergence theorems. Analytic functions, conformal mappings, bilinear transformations; complex integration: Cauchy's integral theorem. Zeros and singularities; Taylor and Laurent's series; Residue theorem.

Section II

Algebra: Basic theory of matrices and determinants; eigen values and eigen vectors; Groups and their elementary properties; subgroups, normal subgroups, cyclic groups, permutation groups; Lagrange's theorem; quotient groups, homomorphism of groups; Cauchy Theorem and p -groups; the structure of groups; Sylow's theorems and their applications; rings, integral domains and fields; ring homomorphism and ideals; polynomial rings and irreducibility criteria; vector space, vector subspace, linear independence of vectors, basis and dimension of a vector space, inner product spaces, orthonormal basis; Gram-Schmidt process, linear transformations.

Section III

Differential Equations & Theory of Numbers: First order ordinary differential equations (ODEs); solution of first order initial value problems; singular solution of first order ODEs; system of linear first order ODEs; method of solution of $dx/P = dy/Q = dz/R$; Total Equation $Pdx + Qdy + Rdz = 0$ and its solution, Existence of solution of PDE's of first order, Lagrange's and Charpit's methods, Classification and solution of second and higher order PDE's with constant coefficient. Divisibility, Prime numbers, Fundamental Theorem of Arithmetic, Radix Diophantine presentation, Linear Equation, Congruences, CRS, RRS and their properties, Fermat's Euler's Theorem with applications, Euler's ϕ functions, Wilson's Theorem, Quadratic residues.

Section IV

Probability and Statistics: Sample space, discrete probability, independent events, Bayes theorem. Random variables and distribution functions (univariate and multivariate); expectation and moments. Independent random variables, marginal and conditional distributions. Characteristic functions, Moment generating functions and probability generating functions. Standard discrete and continuous univariate distributions.

Linear Programming: Linear programming problem and its formulation; graphical method, simplex method, revised simplex method. Transportation Problems, Methods of obtaining initial basic feasible solutions, MODI method. Assignment problems. Sequencing problems- problems with n jobs and 2 machines, problems with n jobs and k machines. Games and Strategies: Two person zero-sum games, Maximin-Minimax Principle, Mixed Strategies, Solution of 2×2 and $m \times n$ games.