Course Outline for Semester VII (Batch 2024)

S. No.	Category	Course Code	Course Title	Credits	L	T	P	S	Hours per week
1	Major	MTHS407MJ	Stochastic Models	4	4	0	0	0	4
		MTHS408MJ	Advanced Calculus	4	4	0	0	0	4
		MTHS411MJ	Mathematical Modelling	4	4	0	0	0	4
		MTHS409MJ	Complex Analysis	4	4	0	0	0	4
		MTHS410MJ	Quality Control & Reliability theory	4	4	0	0	0	4
		MTHS405MJ	Number theory	4	4	0	0	0	4
		MTHS406MJ	Statistical Inference	4	4	0	0	0	4
2	Minor 5		Students to choose	4					

Note: Students have to choose one elective subject from each basket —

Basket 1

1.	**Complex** Analysis (MTHS409MJ)
2.	**Quality Control & Reliability Theory** (MTHS410MJ)

Basket 2.

1.	**Number Theory** (MTHS405MJ)
2.	**Statistical Inference** (MTHS406MJ)

Basket 3.

1.	**Advanced Calculus** (MTHS408MJ))
2.	**Mathematical Modelling** (MTHS411MJ)

Course Title: Stochastic Models	L	T	P	S	Semester: 7th
Course Code: MTHS407MJ	**4**	**x**	**x**	**x**	**Max Marks**: 100
Credits: 4					

Course objective: The main objective of this course is to apprise the students about the existence of several stochastic processes in real life situations and to equip them with the techniques to study their statistical behavior as a sequence of dependent random variables.

Course Outcomes:

1. Describe the techniques of stochastic processes.
2. Apply the concepts and results of stochastic process in the real-life scenario, including competing risks, branching process, Renewal process, Wiener process etc.

Unit I: Definition and examples of stochastic process. Classification of Stochastic process, random walk and gambler's ruin problems, Elementary problems- Random Walk and Gambler's ruin problems Markov chains: Definition and examples of Markov Chain, Transition probability matrix, Stationary distribution and its interpretation. Chapman-Kolmogorov equation, classification of states, communicating classes, recurrence: non-recurrence, Irreducibility.

Unit II: Stationary probability distribution and its applications. Computation of n-step transition probability matrix by spectral representation. Absorption probability and mean time to absorption. Markov processes, Kolmogorov forward and backward equations, Poisson process, pure birth process (arrival distribution), inter arrival distribution, death process (departure distribution), birth-death processes, Yule Furry process (Linear Growth Process), Differential difference & steady state equations for processes, examples based on the concepts. Sickness and marriage models in terms of Markov processes.

Unit III: Renewal processes, renewal function. Elementary renewal theorem and its applications. Galton -Watson branching processes. Definition and examples of discrete time branching process, Probability generating function and its properties, Offspring mean and probability of extinction. Introduction to Brownian motion process and its basic properties. Maximum likelihood estimation of transition probabilities. MLEs of transition intensities, Simulation of a Markov chain and MCMC method.

Unit IV: Stochastic Processes in Queuing Systems: Structure, elements, operating characteristics, classifications and applications, queuing models, single server queuing model ($M/M/1/\infty$) with an infinite and finite ($M/M/1/N$) system capacity, difference equations and steady-state solutions of the model and various probability of busy and idle period, measures of effectiveness, waiting time distribution of M/M/1 model. General Erlang queuing model, M/M/1 queue special cases. Numerical illustrations based on the concepts.

TextBooks/ References:

1. Ross, S. (2005) Introduction to Probability Models,6/e, Academic Press.
2. Bhat, B.R. (2000). Stochastic Models: Analysis and Applications, 2/e, New Age International, India.
3. Medhi, Jyotiprasad (1994): Stochastic Processes, Wiley Eastern Limited, 2/e.
4. Adke, S.R. and Manjunath S.M. (1985). Finite Markov Processes. Wiley Eastern (New Age Publishing)
5. Taylor and Karlin (1984). An Introduction to Stochastic Modeling, Academic Press.

Course Title: Advanced Calculus	L	T	P	S	Semester: 7th
Course Code: MTHS408MJ	4	x	x	x	Max Marks: 100
Credits: 4					

Course objective: To develop a comprehensive understanding of convergence of sequences and series of functions, differentiation and integration of multivariable functions, and their applications in advanced calculus.

Course outcomes: Upon successful completion of the course, students will be able to:

1. Analyze the behavior of sequences and series of functions and determine conditions for uniform convergence.
2. Apply concepts of limits, continuity, differentiability, and extrema to functions of several variables using analytical methods.
3. Evaluate double and triple integrals and apply theorems such as Green's theorem to solve problems involving multiple integrals.

Unit-I: Sequences and series of functions, pointwise and uniform convergence, Cauchy's criterion for uniform convergence, Tests for uniform convergence of sequences and series of functions, properties of uniformly convergent sequences and series, uniform convergence and continuity, uniform convergence and integration.

Unit-II: Functions of several variables: explicit and implicit functions(Definitions), limit and continuity of a function of two variables, algebra of limits, partial derivative of function of two variables, Mean value theorem, sufficient condition for continuity, differentiability of functions of two variables, sufficient condition for differentiability for functions of two variables, algebra of differentiable functions.

Unit-III:, Concept of partial derivatives of higher order, change in the order of partial derivatives, sufficient conditions for equality of f_{xy} and f_{yx}, Young's theorem and Schwarz's theorem, functions of functions, derivative of composite functions (The chain rule), total differentiability, Jacobians: properties and applications in two and three variables, change of variables, Taylor's theorem. Extreme values: Maxima and minima of functions of two variables, Lagrange's method of undetermined multipliers.

Unit-IV: Integration on R^2: Line integrals, sufficient condition for existence and properties of line integrals, Double integrals: Integration over a rectangle, conditions of integrability, Fubini's theorem, Integrability over a bounded domain, calculation of double integral over a closed domain, Green's theorem in R^2, Integration in R^3: Line, surface and volume integrals, Calculation of triple integrals (simple cases).

Text Books/ References

1. S. C. Malik and Savita Arora, Mathematical Analysis, 6th Edition, New Age, 2021.
2. Sudhir R. Ghorpade, B.V. Limaye: A Course in Multivariable Calculus and Analysis, Springer International Edition, 2010.
3. S. Dineen, Functions of two variable, Chapman and Hall, 1995.
4. David V. Widder, Advanced Calculus, 2nd Edition, PHI, New Delhi, Pvt Ltd.
5. James Stewart , Calculus, Early Transcendentals, Seventh Edition Metric Version, Cengage Learning, Inc. 2010.

Course Title: Mathematical Modelling	**L**	**T**	**P**	**S**	**Semester: 7th**
Course Code: MTHS411MJ	**4**	**x**	**x**	**x**	**Max Marks**: 100
Credits: 4					

Course Objectives: To enable the student to formulate mathematical models of real life problems and find their solutions and present application-driven mathematics motivated by problems from within and outside mathematics.

Course Outcomes: After the end of this course students will be able to

1. Understand concept of modelling and simulation.
2. Construct mathematical models of real world problems and solve them using mathematical techniques.
3. Understand and devise models in economics, medicine.

Unit I: Introduction to mathematical modeling, types of modeling, classification of mathematical models, formulation, solution and interpretation of a model.

Unit II: Models in population dynamics, linear growth and decay models, non-linear growth and decay models, continuous population models for single species, delay models, logistic growth model, discrete models, age structured populations.

Unit III: Fibonacci's rabbits, the golden ratio, compartment models, limitations of mathematical models. Mathematical modeling through system of ordinary differential equations, compartment models through system of ODE's.

Unit IV: Modeling in economics, medicine, international trade, gravitation; planetary motion; basic theory of linear difference equations with constant coefficients, mathematical models through difference equations in population dynamics, finance and genetics, modeling through graphs.

TextBooks / References:

1. J. N. Kapur, Mathematical Modelling in Biology and Medicine, New Age International Publishers (2000).
2. Neil Gerschenfeld : The nature of Mathematical modeling, Cambridge University Press, 1999.
3. A. C. Fowler : Mathematical Models in Applied Sciences, Cambridge University Press, 1997.
4. M. R. Cullen, Linear Models in Biology, John Wiley & Sons (1985).

<table>
<tr><td>Course Title: Complex Analysis</td><td>L</td><td>T</td><td>P</td><td>S</td><td>Semester: 7th</td></tr>
<tr><td>Course Code: MTHS409MJ</td><td rowspan="2">4</td><td rowspan="2">x</td><td rowspan="2">x</td><td rowspan="2">x</td><td rowspan="2">Max Marks: 100</td></tr>
<tr><td>Credits: 4</td></tr>
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Course Objective: To develop a strong foundation in complex variable theory by studying analytic functions, contour integration, and series expansions, with applications in science and engineering.

Course Outcome:

1. Learn the basic techniques of contemporary complex analysis as well as applications of these techniques in harmonic analysis, univalent functions theory and special functions.
2. Evaluate integrals along a path, compute the Taylor and Laurent expansions, determine the nature of the singularities and calculating residues.

Unit I: Algebraic and geometric representation of complex numbers, Argand diagram. Modulus, and argument, exponential form of a complex number. De Moivre's theorem and its applications. Complex plane, neighbourhoods, limits, and continuity of complex functions.

Unit II: Differentiability and analyticity, harmonic and conjugate harmonic functions. Cauchy–Riemann equations: necessary and sufficient conditions for analyticity. Harmonic functions and Laplace equation, Milne–Thomson method for constructing analytic functions. Analyticity of elementary functions: exponential, logarithmic, trigonometric, hyperbolic, and inverse functions.

Unit III: Line and contour integrals in the complex plane. Cauchy's theorem and Cauchy's integral formula with applications. Morera's theorem. Liouville's theorem and Fundamental Theorem of Algebra. Maximum Modulus Principle and Schwarz Lemma. Power series representation and radius of convergence.

Unit IV: Taylor and Laurent series expansions and their regions of validity. Zeros of analytic functions and order of zeros. Classification of singularities: removable, poles, and essential. Isolated singular points and behaviour near singularities.Definition and computation of residues at simple and higher-order poles. Residue at infinity and its significance. Cauchy's Residue Theorem and its applications. The argument principle and its applications.

TextBooks / References:

1. Complex Variables and Applications – J. W. Brown and R. V. Churchill
2. Introduction to Complex Analysis (2nd Ed.) – H. A. Priestley, Oxford University Press
3. Complex Variables – Spiegel, Lipschutz, Schiller, and Spellman (Schaum's Outline)
4. Functions of a Complex Variable – John B. Conway
5. Complex Variables: Introduction and Applications (2nd Ed.) – M. J. Ablowitz and A. S. Fokas, Cambridge University Press

Course Title: Quality Control & Reliability	**L**	**T**	**P**	**S**	**Semester: 7th**
Course Code: MTHS410MJ	**4**	**x**	**x**	**x**	**Max Marks**: 100
Credits: 4					

Course Objective: To introduce the key statistical methods used in quality control and reliability analysis, with a focus on their application in industrial processes

Course Outcomes: By the end of this course, students will be able to:

1. Apply statistical and probabilistic methods to monitor and enhance quality in industrial operations and develop control charts for process variables and product attributes.
2. Evaluate product and process quality using standard SQC tools and methods.
3. Design, analyze and compare single and double acceptance sampling plans with performance measures.
4. Compute and interpret system reliability and maintainability using life distributions and system models.

Unit-I: Introduction to quality and dimensions of quality; Definition and objectives of quality control; Statistical quality control (SQC) and its elements; Techniques of SQC; Process control and product control; causes of variation – chance and assignable causes; Shewhart's control charts; Statistical basis of control charts; 3σ and 6σ limits and interpretation; Control charts for variables (X, R and S); Control charts for attributes (np, p and c) charts; Control chart patterns – natural and unnatural variations; Advantages and limitations of SQC.

Unit-II: Introduction to Acceptance Sampling Plans; Types of sampling plans (Single, Double, Rectifying); Implementation and design aspects; Key terms in acceptance sampling; Operating Characteristic (OC) Curve; Producer's and Consumer's Risks; Average Outgoing Quality (AOQ) and AOQ Limit (AOQL); Average Sample Number (ASN); Differences between Single and Double Sampling; Advantages and limitations of various sampling plans.

Unit-III: Introduction to process capability; Concept and need for capability indices; Capability indices (Cp, Cpk and Cpm) – interpretation, limitations and comparison; Estimation of capability indices and confidence intervals for normally distributed characteristics; Concept of Six Sigma.

Unit-IV: Basic reliability concepts; Hazard rate function and its interpretation; Distribution of longevity and its moments; Key theorems in reliability theory; Commonly used lifetime distributions including Exponential, Weibull, Gumbel and Rayleigh; Type I and Type II censored samples; System reliability and hazard rates for systems with independent components arranged in series and parallel configurations.

Text Books/ References:

1. Barlow, R.E. and Proschan, F. (1985). Statistical Theory of Reliability and Life Testing; Holt, Rinehart and Winston.
2. Biswas, S.(1996). Statistical Quality Control, Sampling Inspection and Reliability; New Age International Publishers.

3. Montgomery, D.C. (1985) Introduction to Statistical Quality Control; Wiley
4. Phadke, M.S. (1989) Quality Engineering through Robust Design; Prentice Hall
5. Wetherill, G.B. (1977) Sampling Inspection and Quality Control; Halsted Pres

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<tr><td>Course Title: Number theory</td><td>L</td><td>T</td><td>P</td><td>S</td><td>Semester: 7th</td></tr>
<tr><td>Course Code: MTHS405MJ</td><td rowspan="2">4</td><td rowspan="2">x</td><td rowspan="2">x</td><td rowspan="2">x</td><td rowspan="2">Max Marks: 100</td></tr>
<tr><td>Credits: 4</td></tr>
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Course objectives: To develop a strong foundation in number theory and algebraic structures by exploring divisibility, prime numbers, congruences, and advanced polynomial and quadratic forms, enabling students to analyze and solve theoretical and applied mathematical problems effectively.

Course Outcomes: After the end of this course students will be able to

1. Understand and apply fundamental concepts of divisibility, prime factorization, and linear Diophantine equations in problem-solving.
2. Understand fundamental results on prime numbers, congruences, and number-theoretic theorems such as Fermat's and Euler's, along with their applications.
3. Solve and analyze linear and polynomial congruences using Euler's ϕ-function, Wilson's theorem, and the Chinese Remainder Theorem.
4. Understand and apply advanced concepts of polynomial and quadratic forms, including factor and equivalence theorems, Witt's and Hermite's results, with their mathematical applications.

Unit-I: Divisibility, the division algorithm and its uniqueness, Greatest common divisor and its properties. The Euclidean algorithm, Prime numbers. Euclid's first theorem, Fundamental Theorem of Arithmetic, Divisor of n, Linear Diophantine equations. Necessary and sufficient condition for solvability of linear Diophantine equations, Implement algorithms for divisibility tests, GCD, LCM primality tests and modular exponentiation in Python.

Unit-II: Sequence of primes, Euclid's Second theorem, Infinitude of primes of the form 4n+3 and of the form 6n+5. No polynomial f(x) with integral coefficients can represent primes for all integral values of x or for all sufficiently large x, Fermat Numbers and their properties, There are arbitrary large gaps in the sequence of primes, Congruences, Complete Residue System (CRS), Reduced Residue System (RRS) and their properties, Fermat and Euler's theorems with applications.

Unit-III: Euler's $\varphi-$function with examples, Wilson's theorem and its applications to the solution of the congruence and Solutions of linear congruences, necessary and sufficient conditions for the solution of $a_1x_1 + a_2x_2 + \cdots + a_nx_n \equiv \mathrm{c}(mod m)$, Chinese remained theorem with examples, Congruences with prime power and related results, Lagrange's theorem, viz, the polynomial congruence of degree n has at most n roots.

Unit-IV: Factor theorem and its generalization. equivalence of polynomials, equivalence theorem on the number of solutions of congruences, Quadratic arms over a field of characteristic not equal to 2. Equivalence of quadratic forms, Witt's theorem, Hermite's theorem on the minimum of positive definite quadratic form and its applications.

Text Books/ References

1. W. J. Leveque, Topics in Number Theory, Vol. I-, Dover Publications, 2002.
2. I Niven and H.S Zuckerman, An introduction of the Theory of Numbers, Wiley, 5^{th} Edition 2008.
3. David M. Burton, Elementary Number Theory, McGraw Hill Higher Education, 6^{th} Edition 2005.
4. G.H Hardy and Wright, An introduction to the theory of Numbers, Oxford University Press, 6^{th} Edition 2008.
5. J.P. Serre, A course in Arithmetic, GTM Vol. Springer Verlag 1973.

Course Title: Statistical Inference	**L**	**T**	**P**	**S**	**Semester: 7th**
Course Code: MTHS406MJ	**4**	**x**	**x**	**x**	**Max Marks**: 100
Credits: 4					

Course Objectives: **Course Objectives**: To make students aware of several desired properties of the estimator's, different approaches for parameter estimation (point, as well as, interval) and testing (simple, as well as, composite hypotheses) procedures.

Course Learning Outcomes: After successful completion of this course, student will be able to:

1. Understand concept of order statistics and its applications
2. Recognize basic concepts of statistical inference and understand various properties of estimators.
3. Understand Fisher Information, Lower bounds to variance of estimators, MVUE.
4. Apply various estimation and testing procedures to deal with real life problems.

Unit-1: Order statistics: Definition, derivation of p.d.f. of ith order statistics, for a random sample of size n from a continuous distribution. Density of smallest and largest observations. Derivation of joint p. d. f. of i^{th} and j^{th} order statistics, statement of distribution of the sample range. Distribution of the sample median. Extreme values and their asymptotic distribution (statement only) with applications.

Unit-II: Concept of Statistical inference, sampling method and complete enumeration, Definition of population, parameter, parameter space. The concept of estimation, estimator and estimate. Criteria of a good estimator: unbiasedness, consistency, efficiency and sufficiency. Fisher-Neyman factorization theorem, uniformly minimum variance unbiased estimators

Unit-III: Fisher information matrix, illustration with one and two parameters. Cramer-Rao inequality. Minimum variance bound estimator-examples. Rao-Blackwell Theorem. Estimation Methods: Method of moments, Maximum likelihood estimation, Method of minimum Chi Square, Method of least squares and Interval estimation.

Unit-IV: Hypothesis, Simple and composite hypothesis, critical region, type I and type II errors, Size and power of a test, concept of p-value, level of significance. Definition of Most Powerful (MP) test, Neyman - Pearson (NP) lemma for simple null hypothesis against simple alternative hypothesis (with proof) - Illustrations. Most powerful test. Uniformly most powerful test.

Text Books/ References:

1. Casella G, Berger R. L. (2001). Statistical Inference, 2/e, Cengage Learning Pvt. Ltd.
2. Rohatgi, V. K. and Saleh, A.K. Md. E. (2001). Introduction to Probability and Statistics, John Wiley & Sons, New York.
3. Kale, B.K. & Muralidharan, K. (2015) Parametric Inference: An Introduction, Alpha Science International Ltd.
4. Lehmann, E. L. and Romano, J. (2005). Testing Statistical Hypotheses, Springer
5. Lehmann, E.L. and Casella, G. (1998). Theory of Point Estimation. Springer, New York.