FYUP Course Outline for Semester VII

S. No.	Course Code	Course Title	Credits	Category
1	MTHS400MJ	Regression Theory	4	Major
2	MTHS401MJ	Discrete Mathematics	4	Major
3	MTHS402MJ	Reliability theory	4	Major
4	MTHS403MJ	Topology	4	Major
5	MTHS404MJ	Quality Control	4	Major
6	MTHS405MJ	Number theory	4	Major
7	MTHS406MJ	Statistical Inference	4	Major
8	MTHS400MN	Insurance Mathematics	4	Minor

Note: Students have to choose one elective subject from each basket —

Basket 1

1.	**Discrete Mathematics** (MTHS401MJ)
2.	**Reliability Theory** (MTHS402MJ)

Basket 2.

1.	**Topology** (MTHS403MJ))
2.	**Quality Control** (MTHS404MJ)

Basket 3.

1.	**Number Theory** (MTHS405MJ)
2.	**Statistical Inference** (MTHS406MJ)

Course Title: Regression theory	**L**	**T**	**P**	**S**	**Semester:** 7^{th}
Course Code: MTHS400MJ	**4**	**x**	**x**	**x**	**Max Marks**: 100
Credits: 4					

Course Objectives: This course introduces the principles and methods of statistical modelling for applications across diverse fields, covering both theoretical foundations of linear regression models for analysing real-world datasets

Course Outcomes: By the end of this course, students will be able to:

1. Apply and interpret correlation framework and simple/multiple linear regression models using least squares and maximum likelihood methods.
2. Perform hypothesis testing, construct confidence intervals, and use ANOVA to assess regression models.
3. Diagnose and address multicollinearity, outliers, leverage points, and assumption violations, including design-based ANOVA methods.
4. Formulate and analyze generalized linear models such as logistic and Poisson regression for real-world data.

Unit I: Correlation analysis - conceptual frame work. Methods of studying correlation-scatter diagram, Karl Pearson's correlation coefficient and its significance testing. Simple linear regression model; estimation of the model parameters using least squares estimation method and their properties. Practical examples based on simple regression model.

Unit II: Distribution of response variable and regression coefficient estimators in simple linear regression; unbiased estimator for error variance. Testing of hypotheses and confidence intervals for regression coefficients. Multiple linear regression model; estimation of the parameters using least squares method and their properties; partial and global significance of simple/multiple regression models using analysis of variance; practical examples based on multiple linear regression models.

Unit III: Gauss Markov theorem, variance-covariance matrix of parameter vector; residual analysis and regression diagnostics: detecting and dealing with outliers, hat matrix diagonals (in connection with leverage points). Implication of multicollinearity; diagnostics for multicollinearity: VIF and variance decomposition methods.

Unit IV: Generalized Linear Models- Introduction to GLM: systematic and random components, link functions, maximum likelihood estimation: iteratively re-weighted least squares, logistic regression for binary data, Poisson regression for count data. Applications of binary logistic and count regression models.

Text Books/ Reference:

1. Montgomery, D. C., Peck, E. A. and Vining, G. G. (2012) Introductionto Linear Regression Analysis, Wiley Series in Probability and Statistics, Wiley, United States.
2. Seber G. A. F. and Lee, A. J. (2003) Linear Regression Analysis, WileySeries in Probability and Statistics, Wiley, United States.

3. Draper, N. R. and Smith, H. (1998) Applied Regression Analysis, WileySeries in Probability and Statistics, Wiley, United States.
4. Sengupta, D. and Jammalamadaka, S. R. (2003) Linear Models: An Integrated Approach, World Scientific, Singapore.
5. Vinod, H. D. and Ullah, A. (1981) Recent Advances in Regression Methods, M. Dekker, New York, United State

Course Title: Discrete Mathematics	**L**	**T**	**P**	**S**	**Semester:** 7^{th}
Course Code: MTHS401MJ	**3**	**x**	**2**	**x**	**Max Marks**: 100
Credits: 4					

Course Objectives: The course aims to introduce students to the fundamental concepts of discrete mathematical structures and their applications across various branches of science. It also focuses on developing skills in Python programming for graph modelling, analysis and visualization, enabling students to connect theoretical concepts with practical computational tools.

Course Outcomes: After completing this course:

1. Students will apply Boolean algebra and logic to analyse and simplify logical structures.
2. Students will apply graph concepts to model and analyse real-world problems.
3. Students will analyse graph structures, trees, and matrices to solve connectivity and optimization problems.
4. Students will use Python to model, visualize, and analyse graphs and their matrices using NetworkX and Matplotlib.

Unit 1: Boolean Algebra, Postulates of Boolean Algebra, theorems of Boolean algebra, Sum of product and product of sums, Simplification, NAND and NOR implementation, Posets and Lattices and Hasse Diagrams.

Unit 2: Propositional logic, applications of propositional logic, propositional equivalences, predicates and Quantifiers, First order logic, nested quantifiers, Rules of inference.

Unit 3: History, definitions and terminologies, graph, vertices, edges, degree, paths and cycles, degree sequence of a graph, types of graphs: definition and examples of directed, undirected, bipartite, complete and regular graphs complement and subgraphs, Real-life scenarios and applications of different types of graphs in science, technology and data analysis.

Unit 4: Eulerian and Hamiltonian graph structures, isomorphism in graphs, trees and its properties, rooted and binary trees, spanning trees, minimum spanning trees and Kruskal's Algorithms, coloring in graph structures, Adjacency Matrix, Degree Matrix and Laplacian Matrix associated with graphs.

Practical's:

- Implementation and Visualization of Boolean Algebra and Hasse Diagram using Python
- Visualizing social networks (simple graphs) with NetworkX and Matplotlib.
- Representing graph structure, empty graphs, path, cycle graph, complete graph using Python programming.
- Node and edge deletion
- Python code for adjacency and Laplacian matrix computation of some simple graphs and their eigen values.

Textbooks/ References:

1. C.L. Liu: Elements of Discrete Mathematics, Tata Mc-Graw Hill.
2. Reinhard Diestel, *Graph Theory* (Graduate Texts in Mathematics).
3. Narsingh Deo, *Graph Theory with Applications to Engineering and Computer Science*.
4. Bondy and Murty, *Graph Theory and Applications*.
5. S. Pirzada, An Introduction to Graph Theory, Universities Press, Bangalore, 2012.

Course Title: Reliability theory	**L**	**T**	**P**	**S**	**Semester:** 7^{th}
Course Code: MTHS402MJ	**4**	**x**	**x**	**x**	**Max Marks**: 100
Credits: 4					

Course Objective: This course covers the main statistical methods used in reliability and life data analysis. The main distributions used in reliability data analysis are overviewed. The ageing properties of different distributions are explored. A course in reliability helps in probabilistic modeling of the reliability of systems with multiple components and statistical modeling of reliability of individual components based on lifetime data

Course Outcome: After the end of this course students will be able to

1. Define and interpret key concepts of reliability and failure mechanisms.
2. Model and analyze lifetime data using suitable probability distributions.
3. Evaluate the reliability of different system configurations.
4. Apply reliability logic diagrams and computational methods to assess system performance.

Unit I: Reliability: Definition of Reliability. Component reliability, Hazard rate, derivation of the reliability function in terms of the Hazarad rate, Hazard models. Failures: Causes of failures, types of failures, Bath tub curve, Effect of preventive maintenance. Measures of reliability: mean time to failure and mean time between failures.

Unit II: Life distributions and associated survival, conditional survival and hazard rate functions. Exponential, Weibull, gamma life distributions and estimation of their parameters.

Unit III: Classification of Engineering Systems: Series, parallel, series-parallel, parallel-series and non-series-parallel configurations. Expressions for the reliability of the basic configurations.

Unit IV: Reliability Logic Diagrams: Reliability evaluation of Non-series-parallel configurations: minimal tie-set, minimal cut-set and decomposition methods. Deduction of the minimal cut sets from the minimal path sets. Reliability Evaluation of K-Out-Of-N, Reliability of Standby Systems

TextBooks/ References:

1. Barlow R.E. and Proschan F (1985): Statistical Theory of Reliability and Life Testing, Holt, Rinehart and Winston.
2. Lawless, J.F. (1982) : Statistical Models and Methods of Life Time Data, John Wiley Models, Marcel Dekker.
3. Shaked, M. and : Stochastic Orders & Their Applications, Academic Press. Shanthikumar, J.G. (2007)
4. Nelson, W (1982) : Applied Life Data analysis; John Wiley Stochastic orders and their Application, Academic Press.
5. Zacks, S. (1992) : Introduction to Reliability Analysis: Probability Models and Sta

Course Title: Topology	L	T	P	S	Semester: 7^{th}
Course Code: MTHS403MJ	**4**	**x**	**x**	**x**	**Max Marks**: 100
Credits: 4					

Course Objective: Introduces students the basic fundamentals of Topology and its applications like structural topology optimization.

Course Outcomes: After the end of this course students will be able to

1. A shift in perspective: From measuring distance (Analysis) to studying continuity and closeness through open sets.
2. Fundamental Concepts: Mastery of core definitions: open/closed sets, closure, interior, boundary, bases, subspaces, and continuous functions.
3. Topological Invariants: Understanding properties preserved under homeomorphism: connectedness, compactness, and Hausdorff separation.
4. Ability to apply powerful results like the Heine-Borel Theorem and understand their implications.
5. Building Intuition: Developing a geometric intuition for abstract concepts (e.g., deforming a coffee mug into a doughnut) and recognizing underlying topological structures in various mathematical contexts.

Unit I: Metric spaces: Definition and examples, open sets, completeness, convergence, continuous mapping, completion of a metric space, Cantor's intersection theorem. Contraction mapping,Topological spaces: open sets, closed sets, neighborhoods, bases, subbases, the order topology, product topology on XxY, subspace topology, closed sets and limit points, closures,interiors, continuous functions, homeomorphisms.

Unit II: Product topology, metric topology, order topology. Quotient Topology, Construction of cylinder, cone, Mobius band, torus and Klein Bottle.

Unit III: Connectedness and Compactness: Connected spaces, Connected subspaces of the real line, components and local connectedness, Compact spaces, Heine-Borel Theorem, Local – compactness, limit point compactness

Unit IV: concept of first countability, second countability, separability, Tychnoffs theorem, Lebasgue's covering lemma, Separation Axioms: Hausdorff spaces, Regularity, Normality, Urysohn Lemma, Urysohn Metrization Theorem (statement only), Tietze Extension Theorem.

Textbook/References

1. Topology (2 nd Edition) by James Munkres (Prentice Hall)

2. Principles of Topology by Fred Croom (Cengage Learning)

3. Lecture Notes on Elementary Topology and Geometry by Singer and Thorpe (Springer)

4. Introduction to Topology and Modern Analysis by G. Simmons (McGraw-Hill)

5. General Topology by S. Willard (Addison Wesley)

Course Title: Quality Control	L	T	P	S	Semester: 7^{th}
Course Code: MTHS404MJ	**4**	**x**	**x**	**x**	**Max Marks**: 100
Credits: 4					

Course Objective: To introduce the key statistical methods used in quality control and reliability analysis, with a focus on their application in industrial processes

Course Outcomes: By the end of this course, students will be able to:

1. Apply statistical and probabilistic methods to monitor and enhance quality in industrial operations.
2. Develop and interpret control charts for process variables and product attributes.
3. Evaluate product and process quality using standard SQC tools and methods.
4. Design, analyze and compare single and double acceptance sampling plans with performance measures.
5. Compute and interpret system reliability and maintainability using life distributions and system models.

UNIT-I: Introduction to quality and dimensions of quality; Definition and objectives of quality control; Statistical quality control (SQC) and its elements; Techniques of SQC; Process control and product control; causes of variation – chance and assignable causes; Shewhart's control charts; Statistical basis of control charts; 3σ and 6σ limits and interpretation;

UNIT-II: Control charts for variables (X, R and S); Control charts for attributes (np, p and c) charts; Control chart patterns – natural and unnatural variations; Advantages and limitations of SQC. Introduction to Acceptance Sampling Plans; Key terms in acceptance sampling; Operating Characteristic (OC) Curve; Producer's and Consumer's Risks; Average Outgoing Quality (AOQ) and AOQ Limit (AOQL); Average Sample Number (ASN).

UNIT-III: Single and Double sampling plan, Single sampling plan, Implementation and design aspects, Advantages and limitations of various sampling plans, Differences between Single and Double Sampling. Sequential Sampling plan.

Unit IV: Introduction to process capability; Concept and need for capability indices; Capability indices (Cp, Cpk and Cpm) – interpretation, limitations and comparison; Estimation of capability indices and confidence intervals for normally distributed characteristics; Concept of Six Sigma.

Text Books/ References

1. Barlow, R.E. and Proschan, F. (1985). Statistical Theory of Reliability and Life Testing; Holt, Rinehart and Winston
2. Biswas, S.(1996). Statistical Quality Control, Sampling Inspection and Reliability New Age International Publishers.
3. Montgomery, D.C. (1985) Introduction to Statistical Quality Control; Wiley
4. Ott, E.R. (1975) Process Quality Control; McGraw hill
5. Wetherill, G.B. (1977) Sampling Inspection and Quality Control; Halsted Press.

Course Title: Number theory	**L**	**T**	**P**	**S**	**Semester: 7th**
Course Code: MTHS405MJ	**4**	**x**	**x**	**x**	**Max Marks**: 100
Credits: 4					

Course objectives: To develop a strong foundation in number theory and algebraic structures by exploring divisibility, prime numbers, congruences, and advanced polynomial and quadratic forms, enabling students to analyze and solve theoretical and applied mathematical problems effectively.

Course Outcomes: After the end of this course students will be able to

1. Understand and apply fundamental concepts of divisibility, prime factorization, and linear Diophantine equations in problem-solving.
2. Understand fundamental results on prime numbers, congruences, and number-theoretic theorems such as Fermat's and Euler's, along with their applications.
3. Solve and analyze linear and polynomial congruences using Euler's ϕ-function, Wilson's theorem, and the Chinese Remainder Theorem.
4. Understand and apply advanced concepts of polynomial and quadratic forms, including factor and equivalence theorems, Witt's and Hermite's results, with their mathematical applications.

Unit-I: Divisibility, the division algorithm and its uniqueness, Greatest common divisor and its properties. The Euclidean algorithm, Prime numbers. Euclid's first theorem, Fundamental Theorem of Arithmetic, Divisor of *n*, Linear Diophantine equations. Necessary and sufficient condition for solvability of linear Diophantine equations, Implement algorithms for divisibility tests, GCD, LCM primality tests and modular exponentiation in Python.

Unit-II: Sequence of primes, Euclid's Second theorem, Infinitude of primes of the form 4n+3 and of the form 6n+5. No polynomial f(x) with integral coefficients can represent primes for all integral values of x or for all sufficiently large x, Fermat Numbers and their properties, There are arbitrary large gaps in the sequence of primes, Congruences, Complete Residue System (CRS), Reduced Residue System (RRS) and their properties, Fermat and Euler's theorems with applications.

Unit-III: Euler's $\varphi-$function with examples, Wilson's theorem and its applications to the solution of the congruence and Solutions of linear congruences, necessary and sufficient conditions for the solution of $a_1x_1 + a_2x_2 + \cdots + a_nx_n \equiv c(mod m)$, Chinese remained theorem with examples, Congruences with prime power and related results, Lagrange's theorem, viz, the polynomial congruence of degree n has at most *n* roots.

Unit-IV: Factor theorem and its generalization. equivalence of polynomials, equivalence theorem on the number of solutions of congruences, Quadratic arms over a field of characteristic not equal to 2. Equivalence of quadratic forms, Witt's theorem, Hermite's theorem on the minimum of positive definite quadratic form and its applications.

Text Books/ References

1. W. J. Leveque, Topics in Number Theory, Vol. I-, Dover Publications, 2002.

2. I Niven and H.S Zuckerman, An introduction of the Theory of Numbers, Wiley, 5th Edition 2008.
3. David M. Burton, Elementary Number Theory, McGraw Hill Higher Education, 6th Edition 2005.
4. G.H Hardy and Wright, An introduction to the theory of Numbers, Oxford University Press, 6th Edition 2008.
5. J.P. Serre, A course in Arithmetic, GTM Vol. Springer Verlag 1973.

Course Title: Statistical Inference	**L**	**T**	**P**	**S**	**Semester: 7th**
Course Code: MTHS406MJ	**4**	**x**	**x**	**x**	**Max Marks**: 100
Credits: 4					

Course Objectives: **Course Objectives**: To make students aware of several desired properties of the estimator's, different approaches for parameter estimation (point, as well as, interval) and testing (simple, as well as, composite hypotheses) procedures.

Course Learning Outcomes: After successful completion of this course, student will be able to:

1. Understand concept of order statistics and its applications
2. Recognize basic concepts of statistical inference and understand various properties of estimators.
3. Understand Fisher Information, Lower bounds to variance of estimators, MVUE.
4. Apply various estimation and testing procedures to deal with real life problems.

Unit-1: Order statistics: Definition, derivation of p.d.f. of ith order statistics, for a random sample of size n from a continuous distribution. Density of smallest and largest observations. Derivation of joint p. d. f. of i^{th} and j^{th} order statistics, statement of distribution of the sample range. Distribution of the sample median. Extreme values and their asymptotic distribution (statement only) with applications.

Unit-2: Concept of Statistical inference, sampling method and complete enumeration, Definition of population, parameter, parameter space. The concept of estimation, estimator and estimate. Criteria of a good estimator: unbiasedness, consistency, efficiency and sufficiency. Fisher-Neyman factorization theorem, uniformly minimum variance unbiased estimators

Unit-3: Fisher information matrix, illustration with one and two parameters. Cramer-Rao inequality. Minimum variance bound estimator-examples. Rao-Blackwell Theorem. Estimation Methods: Method of moments, Maximum likelihood estimation, Method of minimum Chi Square, Method of least squares and Interval estimation.

Unit-4: Hypothesis, Simple and composite hypothesis, critical region, type I and type II errors, Size and power of a test, concept of p-value, level of significance. Definition of Most Powerful (MP) test, Neyman - Pearson (NP) lemma for simple null hypothesis against simple alternative hypothesis (with proof) - Illustrations. Most powerful test. Uniformly most powerful test.

Text Books/ References:

1. Casella G, Berger R. L. (2001). Statistical Inference, 2/e, Cengage Learning Pvt. Ltd.
2. Rohatgi, V. K. and Saleh, A.K. Md. E. (2001). Introduction to Probability and Statistics, John Wiley & Sons, New York.
3. Kale, B.K. & Muralidharan, K. (2015) Parametric Inference: An Introduction, Alpha Science International Ltd.
4. Lehmann, E. L. and Romano, J. (2005). Testing Statistical Hypotheses, Springer
5. Lehmann, E.L. and Casella, G. (1998). Theory of Point Estimation. Springer, New York

Course Title: : Insurance Mathematics	**L**	**T**	**P**	**S**	**Semester: 6th**
Course Code: MTHS400MN	**4**	**x**	**x**	**x**	**Max Marks**: 100
Credits: 4					

Course Objectives: To make the student conversant with Construction and use of a life table, various types of life assurance contracts, various types of life annuity contracts and Calculation of Net premiums and reserves

Course Outcomes: On completion of the course the students will be able to

1. Define the different life table function and how to use it
2. Define simple assurance and annuity contracts, and develop formulae for the means and variances of the present values of the payments under these contracts, assuming constant deterministic interest.
3. Describe practical methods of evaluating expected values and variances of the simple contracts defined in objective.
4. Describe and calculate, using ultimate or select mortality, net premiums and net premium reserves of simple insurance contracts.
5. Describe the calculation, using ultimate or select mortality, of net premiums and net premium reserves for increasing and decreasing benefits and annuities.

Unit I: The Life Table : The life table-Constructing a life table-Using the life table-The pattern of human mortality-Life table functions at non-integer ages-uniform distribution of deaths (UDD)- constant force of mortality (CFM)-The general pattern of mortality-Select mortality-Constructing select and ultimate life tables

Unit II: Life Assurance Contracts: Life assurance contracts: Pricing of life insurance contracts, Whole life assurance contracts, Term assurance contracts, Pure endowment contracts, Endowment assurance contracts, Critical illness assurance contracts, Deferred assurance benefits, Mean and Variance of the present value random variable Claim acceleration approximation.

Unit III: Life Annuity Contract: Life annuity contracts: Whole life annuities payable annually in arrears, Whole life annuities payable annually in advance, Temporary annuities payable annually in arrear, Temporary annuities payable annually in advance, Deferred annuities, Deferred annuities-due, Continuous annuities, Immediate annuity, Mean and Variance of the present value random variable approximations.

Unit IV: Net Premiums And Reserves: Net premiums and reserves-The basis-The net premium-The insurer's loss random variable-Reserves- Prospective reserve-Retrospective reserves-Conditions for equality of prospective and retrospective reserves-Net premium reserves-Recursive calculation of reserves.

TextBooks / References:

.1. B H Smith "Contingencies of Value", Harvard University Press, 1988.
2. Alistair Neil "Life Contingencies", Butterworth-Heinemann Ltd; illustrated edition (1977).
3. Griffith Davis "Table of Life Contingencies", Longman & Co, 1825: University of California Library.

4. Micheal M Parmenter, “Theory of Interest and Life contingencies with Pension”, 3rd Edition.
5. Bowers, Newton L et al. – “Actuarial mathematics”. 2nd Edition – Society of Actuaries, 1997.