DEPARTMENT OF ELECTRICAL ENGINEERING
ISLAMIC UNIVERSITY OF SCIENCE & TECHNOLOGY

Tutorial-II	Linear Algebra	B.Tech 4^{th} Sem. EE
Marks 10	Spring-2019	

Questions can be discussed in class if required. Solved submissions would be accepted on 5^{th} August 2019 till 10:00 AM only. Clarity and neatness of submission is important.

1. Find a vector x orthogonal to the row space of A, and a vector y orthogonal to the column space, and a vector z orthogonal to the nullspace:

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 3 \\ 3 & 6 & 4 \end{bmatrix}$$

2. Find all vectors in R^3 that are orthogonal to (1,1,1) and (1,-1,0). Produce an orthonormal basis from these vectors.

3. Find a basis for the orthogonal complement of the row space of A:

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 4 \end{bmatrix}$$

4. Show that $x-y$ is orthogonal to $x+y$ if and only if $\|x\| = \|y\|$.

5. Show that the length of Ax equals the length of A^Tx if $AA^T = A^TA$

6. Find the Projection p of the vector b onto a. Also find $e = b-p$ and check that e is perpendicular to a:

$$b = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, a = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}; b = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, a = \begin{bmatrix} -1 \\ -3 \\ -1 \end{bmatrix}$$

7. Project the vector $b = (1,1)$ onto $a_1 = (1,0)$ and $a_2 = (1,2)$. Let the projections be called p_1 and p_2 respectively. Why doesn't the sum $p_1 + p_2$ equal b?

8. Project $b = (0,3,0)$ onto each of the orthonormal vectors $a_1 = (\frac{2}{3}, \frac{2}{3}, -\frac{1}{3})$ and $a_2 = (-\frac{1}{3}, \frac{2}{3}, \frac{2}{3},)$, and then find its projection p onto the plane of a_1 and a_2.

9. From the nonorthogonal a, b, c find the orthonormal vectors q_1, q_2, q_3

$$a = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, b = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, c = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

10. Factor A into QR with orthonormal vectors in Q

$$A = \begin{bmatrix} 1 & 4 \\ 1 & 0 \end{bmatrix}$$

11. Find an orthonormal set q_1, q_2, q_3 for which q_1, q_2 span the column space of

$$A = \begin{bmatrix} 1 & 1 \\ 2 & -1 \\ -2 & 4 \end{bmatrix}$$

12. Express the Gram-Schmidt orthogonalization of a_1, a_2 as $A = QR$:

$$a_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, a_2 = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$$

13. If Q is an orthogonal matrix, so that $Q^TQ = I$ prove that det(Q) equals +1 or -1.

14. Evaluate det(A) by reducing these matrices to triangular form

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 0 & 4 & 6 \\ 1 & 5 & 8 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 1 & 3 \\ 0 & 4 & 6 \\ 1 & 5 & 9 \end{bmatrix}$$

What are the determinants of B, C, AB, A^TA, C^T

15. Find the eigenvalues and eigenvectors of

$$A = \begin{bmatrix} 3 & 4 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 0 \end{bmatrix}$$

Show that the sum of eigenvalues is equal to the trace of the matrices and the product of the eigenvalues equals the determinant.

16. If a 3 by three upper triangular matrix has diagonal entries 1,2,7, how do you know that it can be diagonalized? What is Λ?

17. Suppose A has eigenvalues 1,2,4. What is the trace of A^2? What is the determinant of $(A^{-1})^T$?

18. Factor these two matrices into $A = S\Lambda S^{-1}$

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$$

19. Find the formula for A^k when

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

20. Prove that $B^{1024} = I$ when

$$B = \begin{bmatrix} 3 & 2 \\ -5 & -3 \end{bmatrix}$$

21. Suppose the rabbit population r and the wolf population w are governed by

$$\dot{r} = 4r - 2w$$
$$\dot{w} = r + w$$

If the initial population of rabbits is 300 and that of wolves is 400, what are the populations at time t?