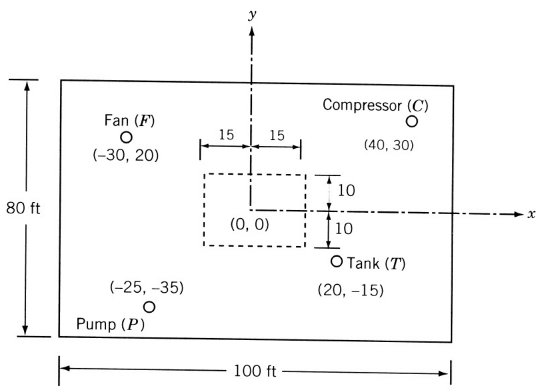
Department of Electrical Engineering
Islamic University of Science & Technology

Tutorial	Optimisation Techniques	8^{th} Sem. EE
Marks 0	Spring-2019	

1. The layout of a processing plant, consisting of a pump (P), a water tank (T), a compressor (C), and a fan (F), is shown in the figure. The locations of the various units, in terms of their x, y coordinates, are also indicated in this figure. It is decided to add a new unit, a heat exchanger (H), to the plant. To avoid congestion, it is decided to locate H within a rectangular area defined by $15 \leq x \leq 15, -10 \leq x \leq 10$. Formulate the problem of finding the location of H to minimize the sum of its x and y distances from the existing units, P, T ,C, and F.



2. Find the minimum of the function $f(x) = 0.65 - \frac{0.75}{1+x^2} - 0.65x\, tan^{-1}(\frac{1}{x})$ using the Newton Raphson method with the starting point $x_1 = 0.1$. Use $\epsilon = 0.01$ for checking convergence.

3. Locate and classify the stationary points of the following functions:
   - $f(x_1, x_2) = x_1^2 + 2x_2^2 - 4x_1 - 2x_1x_2$
   - $f(x_1, x_2) = 10(x_2 - x_1^2)^2 + (1 - x_1)^2$

4. Consider the unconstrained function $f(x_1, x_2) = (x_1^2 - x_2)^2 + x_2^2$. Perform five iterations of a unidirectional search using the golden section search method along $s = (2, 1)^T$ from the point $(-5, 5)^T$ upto the point $(5, 0)^T$.

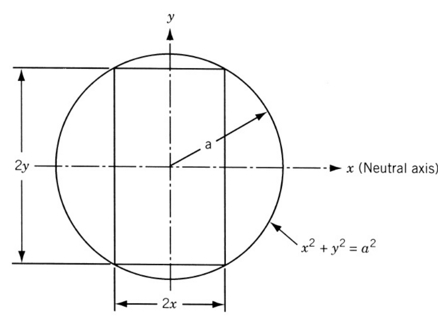
5. Minimise the function given in the previous question using three iterations of the Simplex method. Choose a suitable initial simplex.

6. Given the function $f(x_1, x_2) = 10 - x_1 + x_1x_2 + x_2^2$ and an initial point $x^{(0)} = (2, 4)^T$, minimise analytically using the coordinate descent method and then using the conjugate direction method.

7. Consider the four variable minimisation problem $f(x_1, x_2, x_3, x_4) = (x_1 + 2x_2 - 1)^2 + 5(x_3 - x_4)^2 + (x_2 - 3x_3)^4 + 10(x_1 - x_4)^4$. Perform two iterations of the Cauchy's steepest descent method from the initial point $(2, -1, 0, 1)^T$.

8. Consider the NLP problem of minimising $f(x) = 10 + x^2 - 8x$ subject to the constraint $x \geq 6$. Use the Inverse penalty term with R=1000. Form the penalised function and use exact differentiation to compute the optimal point in the sequence.

9. A beam of uniform rectangular cross section is to be cut from a log having a circular cross section of diameter $2a$. The beam has to be used as a cantilever beam (the length is fixed) to carry a concentrated load at the free end. Find the dimensions of the beam that correspond to the maximum tensile (bending) stress carrying capacity.Use the method of Lagrange multipliers. (Note: *The tensile stress induced in a rectangular beam σ at a distance y from the neutral axis is given by $\frac{\sigma}{y} = \frac{M}{I}$ where M is the bending moment acting and I is the moment of inertia of the cross section about the x axis. If the width and depth of the rectangular beam as shown in the figure are $2x$ and $2y$, respectively, the maximum tensile stress induced is given by $\sigma_{max} = \frac{M}{I}y = \frac{3M}{4xy^2}$*)



10. A glassblower makes glass decanters and glass trays on a weekly basis. Each item requires 1 pound of glass, and the glassblower has 15 pounds of glass available every week. A glass decanter requires 4 hours of labour, a glass tray requires only 1 hour of labour, and the glassblower works 25 hours a week. The profit from a decanter is Rs 50, and the profit from a tray is Rs 10. The glassblower wants to determine the total number of decanters x_1 and trays x_2 that he needs to produce in order to maximize his profit.
    - Formulate an integer programming model for this problem.
    - Solve this model using the branch and bound method.
    - Demonstrate the solution partitioning graphically.